Floor function, rounding, etc.

1. Suppose r is a real number for which

$$\lfloor r + \frac{19}{100} \rfloor + \lfloor r + \frac{20}{100} \rfloor + \lfloor r + \frac{21}{100} \rfloor + \cdots + \lfloor r + \frac{91}{100} \rfloor = 546.$$

Find $\lfloor 100r \rfloor$. (AIME 1991 6)

2. For how many pairs of consecutive integers in $\{1000, 1001, 1002, ..., 2000\}$ is no carrying required when the two integers are added? (AIME 1992 6)

3. Let S be the set of ordered pairs (x, y) such that $0 < x \leq 1$, $0 < y \leq 1$, and $\lfloor \log_2 \left(\frac{1}{x}\right) \rfloor$ and $\lfloor \log_5 \left(\frac{1}{y}\right) \rfloor$ are both even. Given that the area of the graph of S is m/n where m and n are relatively prime positive integers, find $m + n$. (AIME 2004 12)

4. Let $f(n)$ be the integer closest to $\sqrt[4]{n}$. Find $\displaystyle\sum_{k=1}^{1995} \frac{1}{f(k)}$. (AIME 1995 13)

5. Define a positive integer n to be a "factorial tail" if there is some positive integer m such that the base-ten representation of $m!$ ends with exactly n zeros. How many positive integers less than 1992 are NOT factorial tails? (AIME 1992 15)