

Complex numbers

1. Calculate

$$\binom{2007}{0} + \binom{2007}{4} + \binom{2007}{8} + \dots + \binom{2007}{2004}$$

(WOOT 2007-2008)

2. Consider the region A in the complex plane that consists of all points z such that both $z/40$ and $40/\bar{z}$ have real and imaginary parts between 0 and 1, inclusive. What is the integer that is nearest the area of A ? (AIME 1992 10)

3. Suppose that the coefficients of the equation $x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0$ are real and satisfy $0 < a_0 \leq a_1 \leq \dots \leq a_{n-1} \leq 1$. Let z be a complex root of the equation with $|z| \geq 1$. Show that $z^{n+1} = 1$. (AoPS Vol. 2)

4. Let v and w be distinct, randomly chosen roots of the equation $z^{1997} - 1 = 0$. Let m/n be the probability that $\sqrt{2 + \sqrt{3}} \leq |v + w|$, where m and n are relatively prime positive integers. Find $m + n$. (AIME 1997 14)

5. Equilateral triangles ABQ , BCR , CDS , DAP are erected outside of the (convex) quadrilateral $ABCD$. Let X , Y , Z , W be the midpoints of the segments PQ , QR , RS , SP , respectively. Determine the maximum value of

$$\frac{XY + ZW}{AC + BD}$$

(CGMO 2008 4)