1. Find the units digit in the decimal expansion of

$$\left(2008+\sqrt{4032000}\right)^{2000}+\left(2008+\sqrt{4032000}\right)^{2001}+\left(2008+\sqrt{4032000}\right)^{2002}+\cdots$$

$$\cdots+\left(2008+\sqrt{4032000}\right)^{2007}+\left(2008+\sqrt{4032000}\right)^{2008}$$

(2008 iTest # 61)

2. During the car ride home, Michael looks back at his recent math exams. A problem on Michaels calculus mid-term gets him started thinking about a particular quadratic,

$$x^2-sx+p,$$

with roots r_1 and r_2. He notices that

$$r_1+r_2=r_1^2+r_2^2=r_1^3+r_2^3=\cdots=r_1^{2007}+r_2^{2007}.$$

He wonders how often this is the case, and begins exploring other quantities associated with the roots of such a quadratic. He sets out to compute the greatest possible value of

$$\frac{1}{r_1^{2008}}+\frac{1}{r_2^{2008}}.$$

Help Michael by computing this maximum. (2008 iTest # 76)

3. a. Show that for every positive integer n, there exists a polynomial P_n such that

$$t^n+\frac{1}{t^n}=P_n\left(t+\frac{1}{t}\right)$$

for all $t\neq 0$.
b. For what positive integers n do there exist a polynomial Q_n such that

$$t^n-\frac{1}{t^n}=Q_n\left(t-\frac{1}{t}\right)$$

for all $t\neq 0$? (06-07 WOOT)

4. Let $R_n=\left(3+2\sqrt{2}\right)^n+\left(3-2\sqrt{2}\right)^n$, where n is a positive integer. Find the units digit of R_n as a function of n. (AHSME 1990)

5. The sequence a_n is defined by $a_1=a_2=1$, and for $n\geq 1$, $a_{n+2}=a_{n+1}+2a_n$. The sequence b_n is defined by $b_1=1$, $b_2=7$, and for $n\geq 1$, $b_{n+2}=2b_{n+1}+3b_n$. Show that the only integer in both sequences is 1. (USAMO 1973)